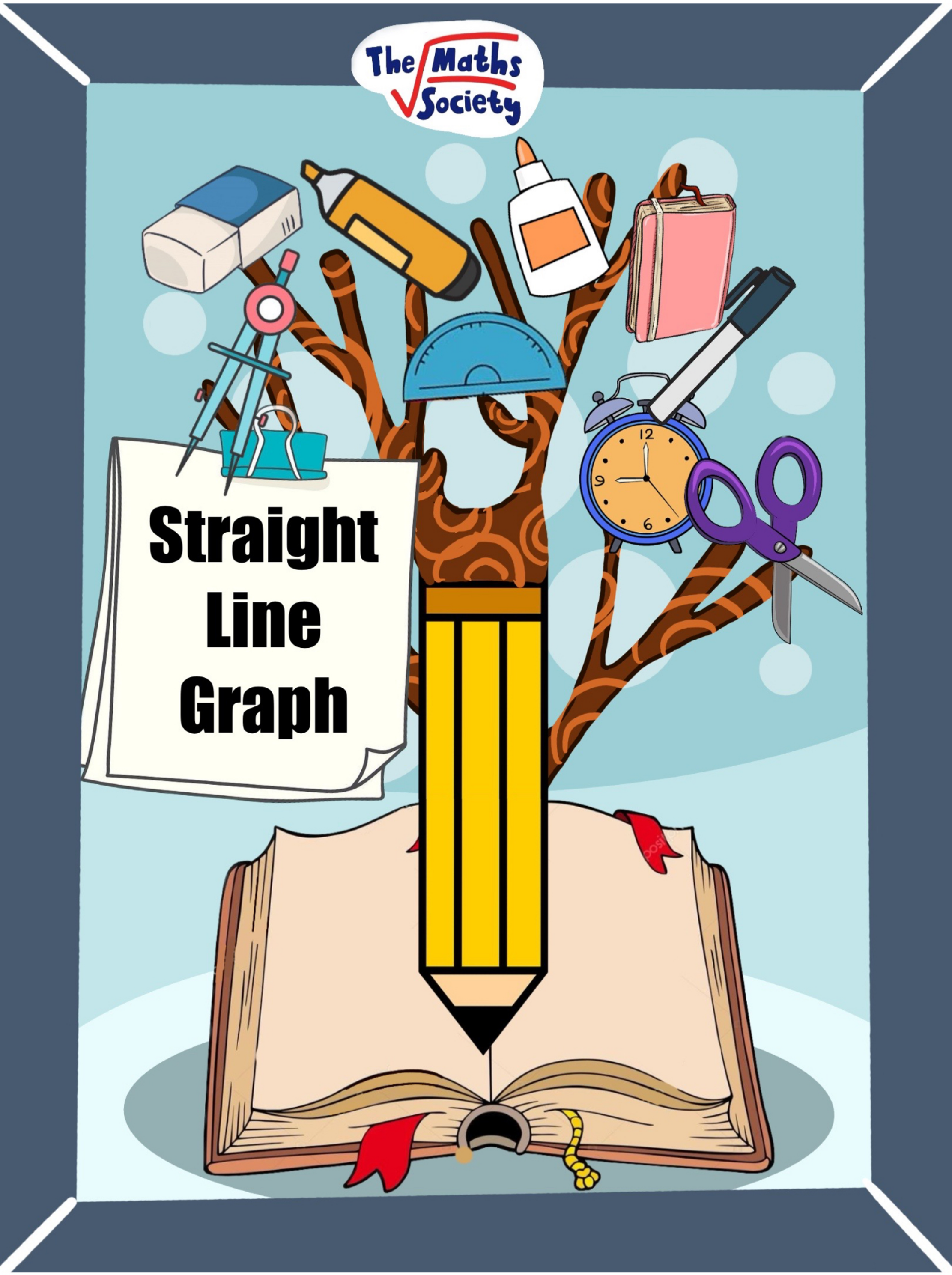

The Maths Society
Straight Line Graph
12
9
6

Straight Line Graph

1. A line ,L, has equation $4x + 5y = 9$. Points A and B have coordinates $(-6, 7)$ and $(1, 9)$ respectively.Find the equation of the line parallel to L which passes through the mid-point of AB.

[3]

$$y = -\frac{4x}{5} + \frac{9}{5}$$

$$m = -4/5$$

$$A(-6, 7) \quad B(1, 9)$$

$$\text{midpt} = \left(\frac{-6+1}{2}, \frac{7+9}{2}\right)$$

$$= (-5/2, 8)$$

$$y = mx + c$$

$$8 = -\frac{4}{\cancel{5}} \times -\frac{\cancel{5}}{\cancel{2}} + c$$

$$8 = 2 + c$$

$$c = 6$$

$$y = -\frac{4}{5}x + 6$$

$$5y = -4x + 30$$

2. Variable x and y are such that when e^{4y} is plotted against x, a straight line of gradient $\frac{2}{5}$, passing through $(10, 2)$ is obtained.

(a) Find y in term of x. [3]

$m = 2/5$

$e^{4y} = \frac{2}{5}x + c$

$2 = \frac{2}{\cancel{5}_1} \times \cancel{10}^2 + C$

$2 = 4 + C$

$C = -2$

$e^{4y} = \frac{2}{5}x - 2$

$4y = \ln\left(\frac{2}{5}x - 2\right)$

$y = \frac{1}{4}\ln\left(\frac{2}{5}x - 2\right)$

(b) Find the value of y when $x = 45$, giving your answer in the form Inp. [2]

$y = \frac{1}{4}\ln\left(\frac{2}{\cancel{5}} \times \cancel{45}^{9} - 2\right)$

$= \frac{1}{4}\ln 16$

$= \ln 2$

(c) Find the value of x for which y can be defined. [1]

$\frac{2}{5}x - 2 > 0$

$\frac{2}{5}x > 2$

$x > 5$

3. Variable x and y are such that when $\sqrt[3]{y}$ is plotted against x^2, a straight line passing through the points $(9, 8)$ and $(16, 1)$ is obtained. Find y as a function of x.

[4]

$$m = \frac{8-1}{9-16} = \frac{7}{-7} = -1$$

$$\sqrt[3]{y} = -1x^2 + c$$

$$1 = -16 + c$$

$$c = 17$$

$$\sqrt[3]{y} = -x^2 + 17$$

$$y = (-x^2 + 17)^3$$

4. The points A, B and C have coordinates $(2, 6), (6, 1)$ and (p, q) respectively. Given that B is the mid-point of AC, find the equation of the line that passes through C and is perpendicular to AB. Give your answer in the form $ax + by = c$, where a, b and c are integers.

[5]

$A = (2, 6)$

$B = (6, 1)$

$C = (p, q)$

midpt $AC = \left(\frac{2+p}{2}, \frac{6+q}{2}\right)$

$\frac{2+p}{2} = 6$

$2 + p = 12$

$p = 10$

$\frac{6+q}{2} = 1$

$6 + q = 2$

$q = -4$

$C = (10, -4)$

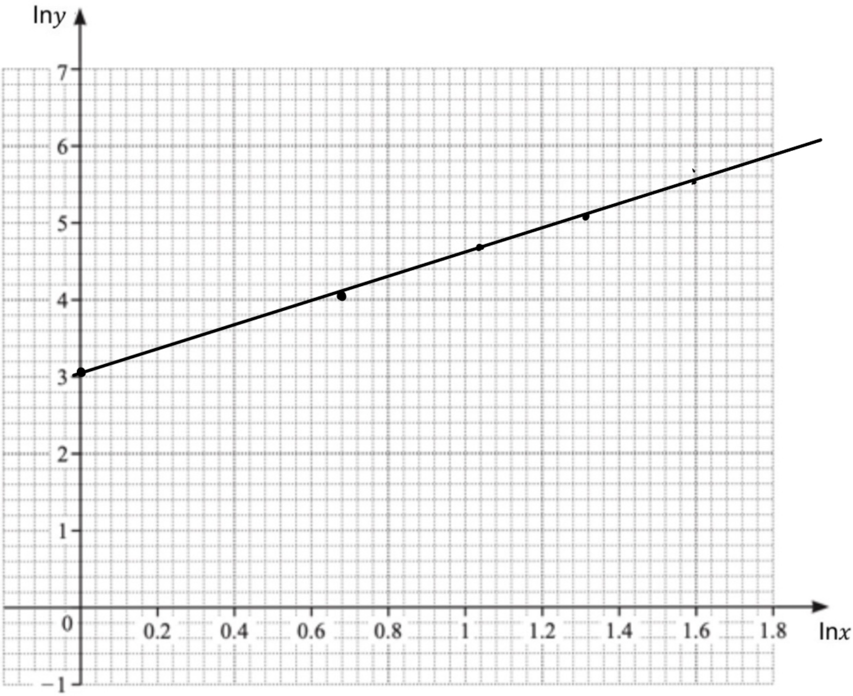
5.

x	1	2	3	4	5
y	20	57	104	160	224
$\ln x$	0	0.6931	1.0986	1.3863	1.6094
$\ln y$	2.9957	4.0431	4.6444	5.0752	5.4116

The table shows value of the variables x and y, which are related by the equation $y = Ax^b$, where A and b are constants.

(a) Use the data to draw a straight line graph of $\ln y$ against $\ln x$.

[3]



(b) Use your graph to estimate the values of A and b. Give your answers correct to 2 significant figures.

[4]

$y = Ax^b$

$\ln y = \ln A + b \ln x$

$\ln A = 3$

$A = e^3$

$= 20.1$

$= 20$

$b = \dfrac{2}{1.2} = 1.67$

$= 1.7$

(c) Use your graph to estimate the value of y when $x = 3.5$.

[2]

$y = Ax^b$

$= 20 \times 3^{1.7}$

$= 129$

6. When In$(y + 2)$ is plotted against x^2 a straight line graph is obtained.The line passes through the points $(2.25, 9.37)$ and $(4.75, 3.92)$. Find y in terms of x.

[5]

$$m = \frac{3.92 - 9.37}{4.75 - 2.25}$$

$$= -2.18$$

$$\ln(y+2) = -2.18x^2 + c$$

$$3.92 = -2.18 \times 4.75 + c$$

$$14.275 = c$$

$$\ln(y+2) = -2.18x^2 + 14.275$$

$$y + 2 = e^{-2.18x^2 + 14.275}$$

$$y = e^{-2.18x^2 + 14.275} - 2$$

7. The perpendicular bisector of the line joining the points $(-\ 3, \frac{2}{3})$ and $(6, -\frac{7}{3})$ passes through the point $(2, k)$.Find the value of k.

[4]

$$m = \frac{-7/3 - 2/3}{6+3} = \frac{-3}{9} = -\frac{1}{3}$$

$$m_{\perp} = 3$$

$$\text{midpt} = \left(\frac{3}{2}, \frac{-5}{6}\right)$$

$$y = 3x + c$$

$$-5/6 = 9/2 + c$$

$$c = -16/3$$

$$y = 3x - 16/3$$

$$k = 6 - 16/3$$

$$= \frac{18-16}{3} = 2/3$$

8. (a) A straight line passes through the points $(4, 23)$ and $(-8, 29)$. Find the point of intersection, P, of this line with the line $y = 2x + 5$.

[5]

$$m = \frac{29-23}{-8-4} = \frac{6}{-12} = -\frac{1}{2}$$

$$y = -\frac{1}{2}x + c$$

$$23 = -2 + c$$

$$c = 25 \qquad y = -\frac{1}{2}x + 25$$

$$2x + 5 = -\frac{1}{2}x + 25$$

$$4x + 10 = -x + 50$$

$$5x = 40$$

$$x = 8$$

$$y = 16 + 5$$

$$= 21$$

$$(8, 21)$$

(b) Find the distance of P from the origin.

[2]

$$\sqrt{64 + 441} = 22.5$$